*Proceedings of **BITCON-2015** Innovations For National Development*
National Conference on : Innovations In Mechanical Engineering For Sustainable Development

Research Paper

ANALYSIS AND MODIFICATION OF ISOTROPIC/ORTHOTROPIC PLATE WITH CENTRAL ELLIPTICAL HOLE SUBJECTED TO IN-PLANE STATIC LOADING FOR REDUCTION OF STRESS CONCENTRATION FACTOR

Shubhrata Nagpal[1], Nitin Jain[2] and Shubhashis Sanyal [3]

Address for Correspondence

[1]Senior Associate Professor, Mechanical Engineering Department, BIT Durg.
[2]Assistant Professor, [3]Professor, Mechanical Engineering Department, NIT Raipur.

ABSTRACT

The stress distribution in isotropic/orthotropic plates with different discontinuities under in-plane static loading has been studied widely. In present paper plate with elliptical hole under in-plane loading has been considered for analysis. Variation in stresses for different a/A ratio (where a is major axis of ellipse and A is width of plate) and b/a ratio (where b is minor axis of ellipse) have been studied. The a/A and b/a has been considered from 0.1 to 0.6 and 0.3 to 0.5 respectively at an interval of 0.1. Stress concentration factor (SCF) has been mitigated by introducing auxiliary circular holes around main elliptical hole. The size and position of auxiliary hole have been optimized for maximum mitigation in SCF. The percentage reduction in SCF after introducing auxiliary holes has been represented graphically. Finite element method has been adapted for analysis. The FEM is carried out using ANSYS software. The results obtained are encouraging in the form of appreciable mitigation of stress concentration.

KEYWORDS Stress Concentration Factor, Elliptical Hole, Auxiliary Holes, Mitigation.

INTRODUCTION

Changes in geometry such as a circular hole or an elliptical hole cause increases in the amount of stress created at these discontinuities. This stress increase is more commonly represented in terms of stress concentration factor. This factor is a ratio between the maximum stresses produced at the discontinuity divided by the nominal stress far away from the hole.

Solution of plate with an elliptical hole was first given by Kolosoff [1]. Hanus [3] formed parameterized geometry models of orthotropic material subjected to uni-axial tension and studied the interaction between elliptical holes and free edges.

Ukadgaonkar [4] has analyzed the stresses in an infinite plate with elliptical hole or crack with tensile stresses. The closed form solutions are given for SCF and SIF (Stress Intensity Factor). It is observed that SCF and SIF depend on the material parameters for anisotropic material, while it is independent of material properties in case of isotropic materials.

Stress concentration around irregular holes using complex variable methods are reported by Simha [5].Conformal mapping is used for evaluation of stress. Nine hole shapes with same area and different perimeter are studied. Irregular holes may change their shape if not their size by exchanging surface energy with strain energy. For complete analysis a global stress analysis is required. Analysis can be extended to a linear visco-elastic material.

Xiwu et al. [6] studied a finite composite plate weakened by elliptical holes under different in-plane loading, treated as an anisotropic multiple connected plates. Using the complex potential method in the plane theory of elasticity of an isotropic body, an analytical solution concerned with stress concentration around an elliptical hole or holes in finite composite fibrous plate is obtained.

The through-thickness variations of stress concentration factors along the wall of elliptic holes in finite thickness plates of isotropic materials subjected to remote tensile stress have been systematically analyzed using the finite element method Chongmin [7]. Nagapl et al. [8] have studied different stresses in plate with circular hole subjected to in-plane loading. Maximum stresses have been reduced by providing auxiliary holes around main hole. The interaction effect of coaxial auxiliary holes on stress concentration around main circular hole for different angular position in an isotropic rectangular plate has been studied.

Chaudhuri et al. [8] have studied stress concentration/intensity around elliptical/circular cylinder shaped surface flaws in cross-ply plates and validity of St. Vansant's principle in the presence of interacting singularities. Stress concentration and stress intensity of external elliptical cylindrical part-through slots, weakening cross-ply plates is evaluated using curvilinear triangular element. Results shows that stress field provides the existence of boundary layer effects and very high stress concentration in the layer comprised of part through slot weakening the cross ply laminate.

In this paper the study of effect of an elliptical hole on the stress distribution of a flat plate has been done. The effect of a/A ratio and b/a ratio on SCF is studied. Rectangular plate with elliptical hole for a/A ratio from 0.1 to 0.6 and a/b from 0.3 to 0.6 has been analyzed for different material.

Nomenclature

L	Length of rectangular plate
A	Width of rectangular plate
a	Major axis of ellipse
b	Minor axis of ellipse
K_t	Stress Concentration Factor , $K_t = \sigma_{max} / \sigma_{nom}$
σ_{max}	Maximum stress on boundary of elliptical hole
σ_{nom}	Nominal stress applied
R1	Radius of first set of auxiliary circular holes
R2	Radius of second set of auxiliary circular holes
δ1	Peripheral distance between elliptical hole and first set of circular hole
δ2	Peripheral distance between elliptical hole and second set of circular hole

PROBLEM DESCRIPTION

Rectangular Plate of size 400 mm*100mm with central elliptical hole has been taken for analysis, Fig.1. In-plane loading is applied on the plate. Elliptical hole of different sizes are considered for a/A = 0.1 to 0.6 and b/a = 0.3 to 0.6 for an interval of 0.1, from convergence test element size of 4 mm and element type of 8 nodded quadratic Plane 82 element has been taken for analysis. The model has been modified by introducing circular holes around the elliptical hole. The model has been analyzed for isotropic as well as orthotropic materials, the properties of materials are given in Table1.

An 8-noded quadrilateral element has been selected. Element length of 4 mm has been chosen after conducting convergence test. The model has been modified for mitigation of SCF by introducing circular holes around the main elliptical hole. The number of auxiliary circular holes increases from two to four.

The equations for a flat plate with a circular hole and an elliptical hole has been compared for the case when the elliptical hole is a circle in order to show that the results are similar enough and equations for elliptical hole can be used for the case when the ellipse becomes a circle.

Table1

		Materials		
Mechanical Properties	**Isotropic Material**	**E-glass/ epoxy**	**Boron/ epoxy**	**Boron/ aluminum**
E_x	39GPa	39 GPa	201 GPa	235 GPa
E_y	-	8.6GPa	21.7GPa	137 GPa
E_z	-	8.6GPa	21.7GPa	137 GPa
G_{xy}	-	3.8GPa	5.4 GPa	47 GPa
G_{yz}	-	3.8GPa	5.4 GPa	47 GPa
G_{zx}	-	3.8GPa	5.4 GPa	47 GPa
ν_{xy}	0.3	0.28	0.17	0.3
ν_{yz}	-	0.28	0.17	0.3
ν_{zx}	-	0.28	0.17	0.3

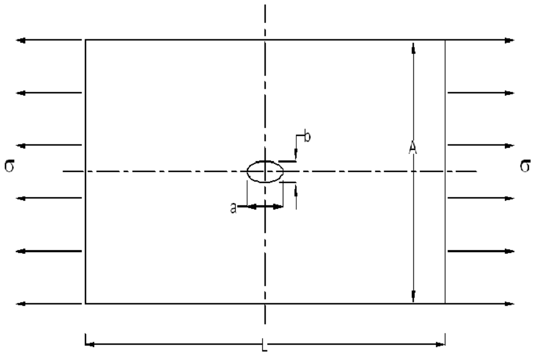

Fig. 1. Rectangular plate with central elliptical hole under in-plane loading

Stress Concentration Factor equations for an elliptical hole in a rectangular plate has been given by R.J.Roark [2]:

$$K = C1 + C2.\left(\frac{2.a}{D}\right) + C3.\left(\frac{2.a}{D}\right)^2 + C4 * \left(\frac{2.a}{D}\right)^3$$

where,

$$C1 = 1 + 2 * \left(\frac{a}{b}\right)$$

$$C2 = -0.351 - 0.021 * \frac{\sqrt{a}}{\sqrt{b}} - 2.483 * \left(\frac{a}{b}\right)$$

$$C3 = 3.621 - 5.183 * \frac{\sqrt{a}}{\sqrt{b}} + 4.494 * \left(\frac{a}{b}\right)$$

$$C4 = -2.270 + 5.204 * \frac{\sqrt{a}}{\sqrt{b}} - 4.011 * \left(\frac{a}{b}\right)$$

……(1.0)

Where,

a = Major axis of ellipse, b = Minor axis of ellipse,

D = Width of the flat plate

K = Stress Concentration Factor for an elliptical hole in a flat plate.

The special case for b = a i.e. for a circular hole, the elliptical hole equation 1.0 yields a SCF of 2.54.

$Kt = 3.00 - 3.13*(D/A) + 3.66*(D/A)^2 - 1.53*(D/A)^3$ (2.0)

Where,

D= Diameter of main hole, A = Width of rectangular plate

For the same plate, using the equation 2.0 for a circular hole, the SCF is 2.50. The difference in SCF values in both the cases is only 1.6%.

The plate has been modified by introducing circular holes around elliptical hole as shown in Fig.2. The SCF around elliptical holes has been analyzed for different sizes and position of circular holes. The diameter of holes is considered as half of minor axis of the ellipse and equal to the minor axis of the ellipse. Plate has been analyzed for both the size of circular hole considered by taking the peripheral distance between the holes varied from half of the diameter of the hole to the twice of diameter of hole.

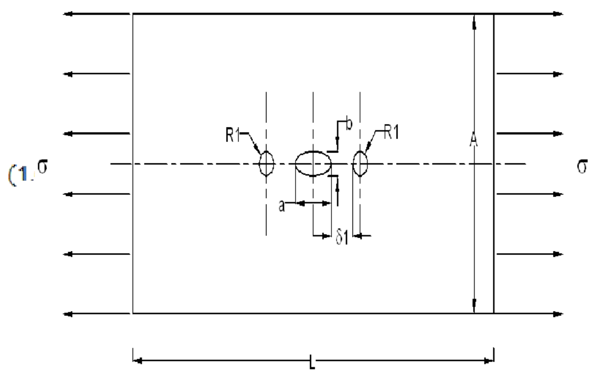

Fig. 2. Rectangular plate with central elliptical hole and two auxiliary circular holes, Model A

RESULT AND DISCUSSION

Fig.3. shows the variation in stress for different geometric combinations of elliptical hole. Maximum stress in x direction is maximum for isotropic material in all cases considered and is minimum for boron/aluminium. As b/a ratio increases the sharpness of ellipse reduces and maximum stress also reduces for all a/A ratios for all materials. The trend of curve is same for all the materials and all a/A ratios.

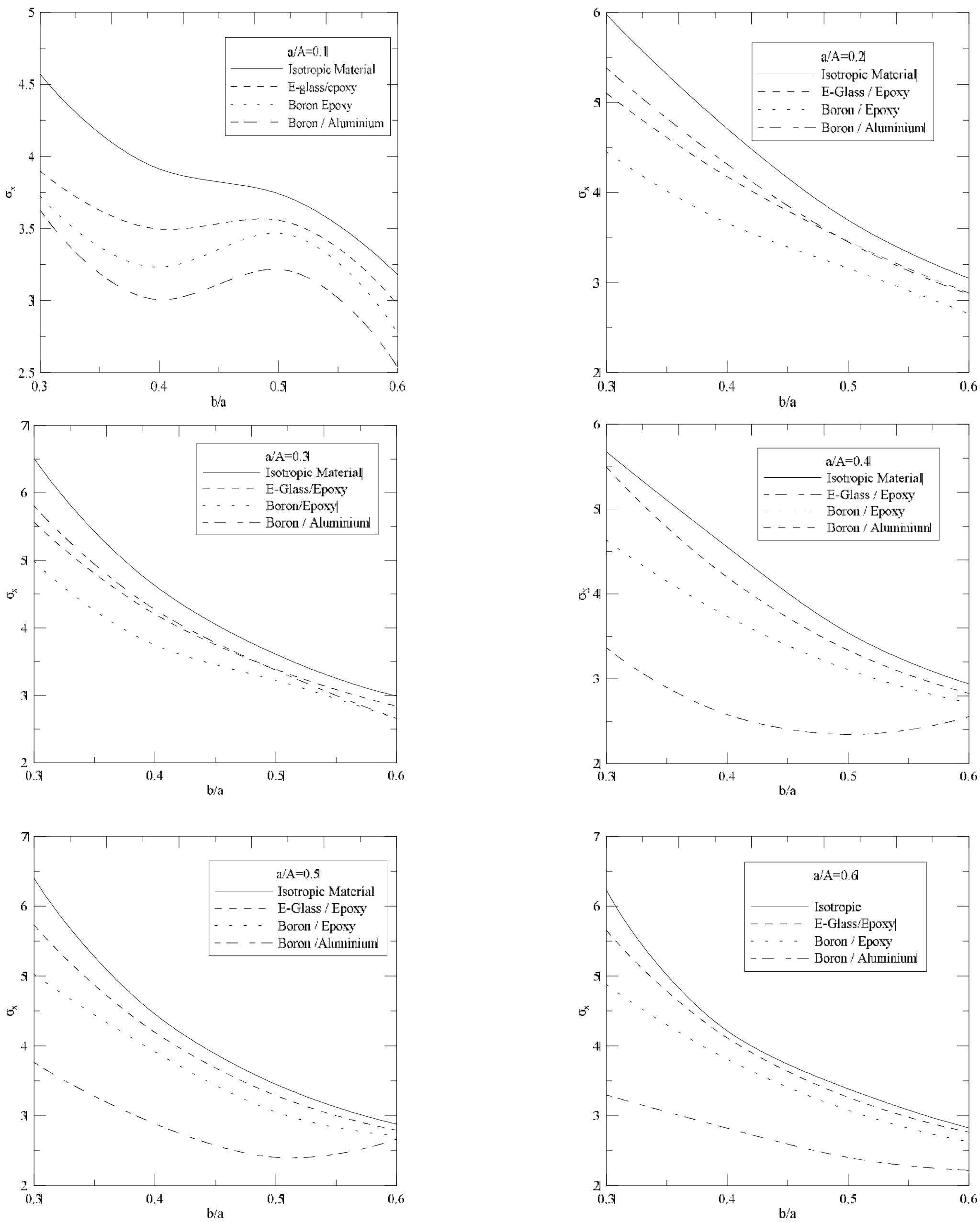

Fig. 3. σ_X Vs b/a for different a/A ratio of all materials

The auxiliary coaxial circular holes are provided around elliptical hole. The variation in SCF has been reported for different a/A ratio and b/a ratio. The variation in SCF has been tabulated for different size and position of auxiliary holes for a/A equal to 0.3 to 0.6 in Table 2 to Table 5.

Table.2. Variation in SCF for different R1 and δ1 with a/A = 0.3

b/a	δ1	R1	SCF
0.3	0.25* b	0.25*b	1.79
0.4			1.58
0.5			1.83
0.6			1.80
0.3	0.5*b	0.25*b	1.45
0.4			1.58
0.5			1.75
0.6			1.82
0.3	**b**	**0.25*b**	1.43
0.4			1.58
0.5			1.70
0.6			1.77
0.3	0.25* b	0.5*b	1.63
0.4			1.43
0.5			1.64
0.6			1.92
0.3	0.5*b	0..5*b	1.80
0.4			1.41
0.5			1.65
0.6			1.89
0.3	b	0.5*b	1.77
0.4			1.44
0.5			1.72
0.6			2.10

Table.3. Variation in SCF for different R1 and δ1 with a/A = 0.4

b/a	δ1	R1	SCF
0.3 0.4 0.5 0.6	0.25* b	0.25*b	1.55 1.59 1.99 2.00
0.3 0.4 0.5 0.6	0.5*b	0.25*b	1.46 1.70 1.87 1.76
0.3 0.4 0.5 0.6	**b**	**0.25*b**	1.50 1.70 1.89 1.86
0.3 0.4 0.5 0.6	0.5*b	0..5*b	1.40 1.47 1.64 1.72
0.3 0.4 0.5 0.6	b	0.5*b	1.43 1.54 1.64 1.82

Table 4. Variation in SCF for different R1 and δ1 with a/A = 0.5

b/a	δ1	R1	SCF
0.3 0.4 0.5 0.6	0.25* b	0.25*b	1.43 1.55 2.00 1.75
0.3 0.4 0.5 0.6	0.5*b	0.25*b	1.55 1.53 1.92 1.74
0.3 0.4 0.5 0.6	**b**	**0.25*b**	1.43 1.58 1.86 1.75
0.3 0.4 0.5 0.6	0.25* b	0.5*b	1.48 1.50 1.73 1.60
0.3 0.4 0.5 0.6	0.5*b	0.5*b	1.56 1.52 1.75 1.68
0.3 0.4 0.5 0.6	b	0.5*b	1.76 1.58 1.72 1.72

Table 5. Variation in SCF for different R1 and δ1 with a/A = 0.6

b/a	δ1	R1	SCF
0.3 0.4 0.5 0.6	0.25* b	0.25*b	1.80 2.00 1.75 1.35
0.3 0.4 0.5 0.6	0.5*b	0.25*b	1.82 1.76 1.74 1.72
0.3 0.4 0.5 0.6	**b**	**0.25*b**	1.77 1.86 1.75 1.74
0.3 0.4 0.5 0.6	0.25* b	0.5*b	1.92 1.61 1.60 1.59
0.3 0.4 0.5 0.6	0.5*b	0..5*b	1.89 1.72 1.68 1.66
0.3 0.4 0.5 0.6	b	0.5*b	2.10 1.82 1.72 1.76

CONCLUSIONS

The size of elliptical hole b/a ratio affects the SCF around the elliptical hole. As the size of elliptical hole with respect to width of plate, a/A increases the stresses also increase. As b/a ratio increases the stresses decreases from which it is concluded that as the sharpness of ellipse reduces the stresses decreases. The stresses reported are more in case of ellipse as compared to circle for same loading conditions. The area reduction method can be applied for mitigation of SCF around the ellipse. The introduction of auxiliary holes reduces the SCF on elliptical hole as it causes the smooth stiffness changes along the direction of acting force is beneficial to the decrease of the stress concentration.

The distance and size of circular holes effects the mitigation of SCF around the elliptical hole. The increase of number of auxiliary circular holes around the ellipse along the direction of applied force is beneficial for the mitigation of stress concentration.

The maximum reduction in SCF has been reported when the auxiliary holes are placed near to the elliptical hole and are of equal to minor axis of the ellipse.

ACKNOWLEDGEMENT

The authors acknowledge the institute authorities for supporting the present work to be carried out in the institute.

REFERENCE

[1] Kolosoff , C.E.Inglis , “ Solution of plate with an elliptical hole ” , Transaction of Institute of Noval Arch. , London , 1913.

[2] Roark R. J., “ Formulas for stress and strain ”, 4th edition, McGraw Hill,1965.

[3] J.B Hanus, C.P Burger, “Stress Concentration factors for elliptical holes near an edge”, Experimental Mechanics, Vol. 21, 1981.

[4] V. G. Ukadgaonkar, Awasare P.J., “A novel method of stress analysis of an infinite anisotropic plate with elliptical hole or crack with uniform tensile stress”, Journal of Institution of Engineers (India), 75:53-5, MC, 1994.

[5] K R Y Simha, SS Mahapatra “Stress Concentration around irregular holes using complex variable method”, Sadhana, 23,1994.

[6] X. Xiwu, S. Liangxin, and F. Xuqi, “Stress concentration of finite composite laminates weakened by multiple elliptical holes,” International Journal of Solids Structures, Vol.32(20), pp. 3001-3014, 1995.

[7] Chongmin She, Wanlin Guo, “Three-dimensional stress concentrations at elliptic holes in elastic isotropic plates subjected to tensile stress”, International Journal of Fatigue, Vol.29, pp 330–335, 2007.

[8] Nagpal Shubhrata, Sanyal S., Jain N.K., “Interaction Effect of Auxiliary holes for Mitigation of Stress Concentration in Isotropic Plate with Central Circular Hole Subjected to in Plane Loading”, International Journal of Mechanics and Solids, ISDN 0973-1881 Volume- 6 Number-2, pp 149-156, 2011.

[9] Chauduri R, Oktem A, Soares C. “Stress concentration / intensity around elliptical / circular cylinder shaped surface flaws in cross-ply plates and validity of St. Vesant’s principle in the presence of interacting singularities”, Applied Mathematical Modeling; 37(3): pp 1362–1377, 2013.

Note: This Paper/Article is scrutinised and reviewed by Scientific Committee, BITCON-2015, BIT, Durg, CG, India